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A SENSITIVE CRYOGENIC ACCELEROMETER*

ABSTRACT

LLAMA (for Low-Level Acceleration Measurement Apparatus) is an investigation of techniques for the measurement of acceleration in the range 10^{-6} to 10^{-9} g. In developing an experimental apparatus, the approach was to seek unconventional mechanizations of the various components of a simple, linear, single-axis accelerometer, with considerable emphasis on system compatibility and with explicit provision of a means for calibration of the instrument. The resultant design choices are briefly outlined. Problems encountered in the design of a superconducting (Meissner effect) test mass suspension are described, together with results obtained from a feasibility demonstration model of the suspension. A laser interferometer is being developed for the displacement detector and design principles for such a device are discussed. Restoring forces are generated by means of small coils within the suspension, and calibration is effected by means of controlled radiation pressure.

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RE-13

A SENSITIVE CRYOGENIC ACCELEROMETER*

INTRODUCTION

As shown in Figure 1, an elementary linear accelerometer consists of five major components. Very stringent performance requirements are imposed on these components, if the system is to operate satisfactorily at very low acceleration levels. In particular, the test mass suspension must offer a virtually force-free environment with respect to translation along the sensitive axis of the instrument. The displacement detector sensitivity is determined primarily by the bandwidth required, because the test mass moves very slowly at the acceleration levels under consideration: for instance, starting from rest at 10^{-9} g, a particle moves through 0.005 micron during the first second. For many applications, displacements equal to a small fraction of the wavelength of light must be detected.

In view of the difficulty of reaching these performance objectives for the various components, the question of their compatibility in the overall system must be considered at an early stage in the design. If the instrument is to be capable of absolute measurement, some means for calibration must be provided, because of the problem of generating known test accelerations below 10^{-6} g.

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TEST MASS SUSPENSION

Almost any mechanical support displays hysteretic effects at the force levels under consideration, so various types of electromagnetic field suspension were investigated. Of these, the cryomagnetic (Meissner effect) suspension offered the lowest residual forces along the sensitive axis, and also appeared to be the easiest to mechanize.

It is well known that superconducting materials, being perfectly diamagnetic, are repelled by a magnetic field. This property is used in the cryogenic gyro, in which the inertial element is a superconducting sphere, supported by the field of currents flowing in adjacent coils. In LLAMA, on the other hand, the test mass is a small permanent magnet, which is stably suspended over a superconducting surface. This is an adaptation of the famous 'floating magnet' experiment of Arkadiev (Ref. 1).

The phenomenon involved in this suspension is perhaps best understood by considering the case in which the superconducting surface is an infinite horizontal plane. Perfect diamagnetism means that the field of the small magnet cannot penetrate the superconductor, and hence there can be no component of field at the surface in the normal direction. It is easy to see that the resulting field distribution is identical to that which would be created by the magnet plus a positive image magnet below the surface. The float height is thus equal to half the separation at which the repulsion of two such magnets is just equal to the weight of one of them.

Since the image of course moves with the magnet, there can be no horizontal component of force from the superconductor. If, however, the superconducting plane be finite in extent, the field spills over at the edge, producing a force which tends to draw the magnet towards the nearest boundary. In other words, a finite planar suspension is slightly unstable with respect to horizontal motion.

In order to make a device which is sensitive only along a single axis, a superconducting tube rather than a plane is used in the LLAMA suspension. The magnet then encounters a restoring force on displacement in any direction away from the axis of the tube. Because of the edge effect mentioned above, the suspension is unstable with respect to axial displacement of the test mass away from the center of the tube, but this instability can be reduced arbitrarily by lengthening the cylinder, or compensated as part of the design of the restoring force generator (see below).

In practice, it is necessary that the superconductor be singly-connected, since otherwise movement of the magnet would generate persistent supercurrents, giving the suspension itself a magnetic moment. This effect can be used to provide end effect compensation, but it complicates the design. In the experimental apparatus, the superconducting tube therefore has a narrow axial slit along the top.

The LLAMA suspension dewar is shown in Figure 2 and, in cross-section, in Figure 3. A thin-walled copper cylinder (O. D. 2.5 cm) was inserted lengthwise in a hollow copper block, attached to the bottom of the inner vessel of the dewar. The superconductor was a sheet of niobium, wrapped around the outside of this cylinder, so that it was in contact with liquid helium when the dewar was filled. The ends of the cylinder were open to the dewar vacuum, and the magnet, when inserted, floated just below the center line.

A problem with all cryomagnetic suspensions is the phenomenon of flux-trapping. The transition temperatures of all superconductors decrease monotonically with applied magnetic field, so that, if the metal be cooled in a field which exhibits a local maximum of intensity, this region will remain normal after the rest of the material is superconducting. Since magnetic field cannot penetrate the superconductor, any flux passing through the normal region will be irrevocably trapped until the temperature is raised.

The implication of this effect in the present context is that the magnet must be external to the cylinder at the time the dewar is filled with liquid helium. The magnet was therefore stored in the room-temperature, evacuated antechamber shown at the left in Figure 3. When the niobium was superconducting, the magnet was inserted into the suspension by means of a lucite spoon, which could be manipulated from outside the dewar via a vacuum feed-through.

The dewar was equipped with windows opposite the ends of the cylinder, which were used for observation of the test mass and for access by the displacement detection system. The magnet is shown floating inside the suspension in Figure 4; the bright annulus seen in this photograph is one of two aluminum discs which were attached to the magnet to simulate the weight of mirrors which are part of the displacement detector (see below).

In order to overcome the axial instability due to end effects, small coaxial coils were placed inside the copper tube, near each end. When these were fed in series with a suitable current, in the repulsive sense, the magnet was found to settle near the center of the tube. By reducing the current in these coils to the minimum which would give stability, it was possible to produce a nearly force free region in the center of the suspension.

The copper cylinder between the magnet and the superconductor produced eddy-current damping of all modes of oscillation of the test mass. The time constant for axial oscillation was found to be about three seconds.

In addition to compensating for end effects, the coils inside the suspension, when fed with a differential current controlled by the output of the displacement detector, provided the restoring force required for operation of the apparatus as an accelerometer.

EVALUATION OF SUSPENSION

A theoretical analysis has been made of the field distribution inside the suspension, (Ref. 2) the object being to estimate the ultimate sensitivity which can be reached with such a system. Apart from the end effects, the limiting factor appears to be surface roughness and distortion of the superconductor. This effect is not as important in LLAMA as in some other field-supported devices, because the inertial element floats at a height of about 1 cm as compared with the clearances of the order of 0.01 cm which are used, for instance, in the electrostatic gyro. It is expected that, with careful construction, the instrument might be useful to the 10^{-9} - 10^{-10} g range.

The analysis indicates that the axial force inside the superconducting tube is given by

$$F_x = \alpha \sinh 2 cx \quad (1)$$

where,

F_x = force on magnet at axial displacement x from null.

α = constant - depends on geometry of tube and magnet and on the pole strength of the magnet.

c = constant - depends on the radius of the tube.

The force on the magnet due to the coils is found to be,

$$\begin{aligned} F_{\text{coils}} &= \frac{-\beta}{2} \left[(I + \Delta I) e^{cx} - (I - \Delta I) e^{-cx} \right] \\ &= -\beta I \sinh cx - \beta \Delta I \cosh cx \end{aligned} \quad (2)$$

where F_{coils} = force on magnet due to coils when magnet is at displacement x from null.

β = constant - depends on the geometry of the coils and the magnet.

c = constant as in Eq. (1).

I = standing current in each coil.

$2 \Delta I$ = differential current in coils.

A sketch of the uncompensated axial suspension force and the compensating effect of the coils for small displacements from null is shown in Figure 5. For a suitably chosen current in the coils a small region exists around null where the magnet is axially stable; outside this region the suspension is unstable. The length of this stable region (taken as the distance between the peaks e.g. $p-p'$ in Figure 5) corresponding to a desired spring constant at null is shown in Figure 6. The smaller the desired spring constant, the shorter the stable region and hence the more sensitive the displacement detector has to be. One way of lengthening the stable region for a given spring constant is to reduce the inherent suspension spring constant as illustrated by the dotted curves in Figure 6.

In the absence of the interferometer (see below) an optical displacement detector using a spot occultation technique, shown in Figure 7, was constructed and attached to the dewar (Ref. 3). The linear range of this detector was approximately 1 mm.

The effective spring constant in a 1 mm region around null was determined by measuring the natural frequency of oscillation of the magnet. Figure 8 shows a plot of spring constant as a function of coil current. By extrapolation, the inherent suspension spring constant is found to be approximately -15 dynes/cm. Because of the general vibrations of the surroundings, the lowest value of spring constant measured was 2 dynes/cm.

A summary of results of tests performed in the preliminary evaluation of the suspension is given below:

Initial inherent suspension spring constant.	-15 dynes/cm
Lowest measured spring constant.	2 dynes/cm
Natural axial damping time constant	3 sec
Threshold sensitivity as an elementary accelerometer	10^{-5} g

PERFORMANCE OF PRELIMINARY ACCELEROMETER

By using the output of the spot occultation detector to control the differential current in the restoring coils, the closed loop behavior of this rather crude version of the accelerometer was investigated.

The overall block diagram for the accelerometer is shown in Figure 9. If the displacements are small the elements in the diagram can be linearized and the loop is seen to be unstable. This observation was substantiated by experiment and the loop was stabilized with the aid of some lead compensation. However, just before the loop did stabilize, an oscillation about the transverse horizontal axis of the magnet was excited as shown in Figure 10. This excited mode was due to the detector being sensitive to rotation of the magnet and was eliminated by suitable filtering.

The accelerometer in this crude form detected tilts equivalent to 10^{-5} g.

The excitation of a rotary oscillation is at least partly due to the magnetic axis of the test mass not coinciding with its principal axis. This causes the axial restoring force from the coils to apply a torque to the magnet. If the displacement detector is sensitive to rotation, an oscillatory condition can exist.

Figure 11 shows the coupling of the oscillatory mode for a given detector rotational sensitivity α_L and a torque/differential current ratio K. The only place where compensation is feasible is in the servo block $AG_2(s)$, in the main accelerometer loop.

SENSITIVE DISPLACEMENT DETECTOR

As mentioned earlier, very small axial displacements, on the order of a fraction of a micron, of the magnet must be detected in order to preserve an adequate bandwidth at low acceleration levels. Another reason for desiring high sensitivity is that high axial spring constant can be tolerated if the displacement is always small.

The interferometric technique was found to offer both high sensitivity and compatibility with the rest of the system. The magnet is easily accessible by the interferometer light beams through the windows on either side of the dewar. The principal mirrors of the interferometer are attached to each end of the magnet as shown in Figure 12.

To obtain more sensitivity and to simplify the method of readout, a Twyman-Green configuration is chosen in which the fringe pattern is a uniform field, modulated in intensity by the axial displacement of the magnet. The advantage of having both principal mirrors on the magnet is that this doubles the sensitivity compared to a normal interferometer and also provides a means for making the displacement detector insensitive to rotation of the magnet about its transverse axes. Another advantage is that both principal mirrors are subjected to the same environmental disturbances.

Normally, when the magnet rotates a little about a transverse axis the interfering beams emerging from the beam splitter no longer coincide and the common overlap area displays the formation of an optical wedge. However, if one principal mirror is optically completely reverted the interfering beams will move together and in addition no wedge will be formed. An equivalent effect can be achieved by placing a three-mirror dove prism, or reversion prism, in each leg of the interferometer, the axes of the prisms being 90° apart.

The use of retro-reflectors (or corner cubes) instead of optical flats at either end of the magnet to eliminate rotation effects is also being investigated. Because of weight and float height limitations, the retro-reflectors have to be small, very light and yet accurate.

The illumination of the interferometer in this configuration must be by a highly collimated beam having a narrow spectral bandwidth. The spatial and temporal coherence of a gas laser beam is very suitable for this application.

After interference, the light emerging from the beam splitter shown in Figure 12 is gathered by a lens and integrated by a photodetector. The output of the photodetector controls the differential current in the restoring coils. The position of the magnet is adjusted so that the output

intensity is half way between that corresponding to constructive and destructive interference. Although this output changes with displacement, it does not carry information about the direction of the displacement. This is of no consequence since if the feedback has the wrong sign the magnet will move a distance $\frac{\lambda_d}{2}$, causing a sign reversal, and the magnet will stabilize around this new position. λ_d is one quarter of the wavelength of the light beam and is the distance the magnet moves for the output intensity to vary through a complete cycle.

CALIBRATION

As mentioned earlier, a primary consideration in the design of LLAMA is that it should not require an externally applied acceleration for calibration, since known accelerations in the range of interest are difficult to generate in the laboratory. 10^{-6} g corresponds to a table tilt angle in a 1 g-field of 0.2 arc seconds and 10^{-9} g corresponds to 0.0002 arc seconds.

The alternative is, of course, to apply a known small force directly to the test mass. The most convenient source of such a force (Ref. 3), in this application, is radiation pressure, because of its readily calculable nature.

Since the test mass environment is a very good vacuum, due to cryopumping, it is suitable for the application of radiation pressure. Any heating of the test mass by the high intensity beam of light will increase the rate of consumption of liquid helium whereas, in the case of a superconducting test mass, the heating effect of the beam, enhanced by the thermal isolation, would destroy its superconductivity.

Since table tilt calibration methods in a 1 g-field can be used down to 10^{-6} g (or 0.2 arc seconds) the maximum force needed from radiation pressure, at this acceleration level, is 10^{-3} dynes applied to the 1 gram test mass. The beam power required to exert 10^{-3} dynes on a perfectly reflecting surface is 1.5 watts.

Various sources of high intensity radiation have been investigated and a 100 watt mercury arc lamp was found to be adequate.

A simple optical configuration was set up to gather the light and concentrate it on an area equivalent to that of the mirror attached to the magnet. The total beam angle was kept below 12° because of the geometry of the present suspension. The power received as measured by a flat response detector was 3.5 watts.

The complete LLAMA system is shown in Figure 13. The calibration light beam, when used, is isolated from the interferometer beam by the use of a dichroic mirror (dielectric rejection filter) in one leg, as shown. The spectral filtering required is relatively simple because of the wide spectral separation of the two sources used.

FUNDAMENTAL LIMITS TO SENSITIVITY

The major advantage of the LLAMA suspension is that there is no mechanism present which might cause "threshold," so that linearity of the system should be preserved down to very low levels.

Distortion or surface roughness of the superconductor may produce conservative axial forces on the test mass, but the large float height obtained makes this problem much less serious than in, for instance, an electrostatic suspension. In other words, any surface roughness effects are averaged over distances of the order of centimetres, so that, with care in manufacture, this effect should not be observable down to extremely low levels.

Another limitation is oscillation of the test mass caused by thermal fluctuations, small changes in the magnetic pole strength, etc. The importance of this may be estimated by means of the equipartition theorem.

The acceleration threshold due to this effect turns out to be inversely proportional to the natural period of axial oscillation of the system. If a natural period of 8 hours were acceptable, it might be possible to measure accelerations down to about 10^{-12} cm/sec². Of course, this would require a very quiet location, such as may be obtained in orbit. With great care in the control of any external magnetic fields (for instance by external superconductive shielding), and using a heavier test mass, it is perhaps possible to improve this threshold by a factor of about 10^4 ; this would require a natural period of the order of one year.

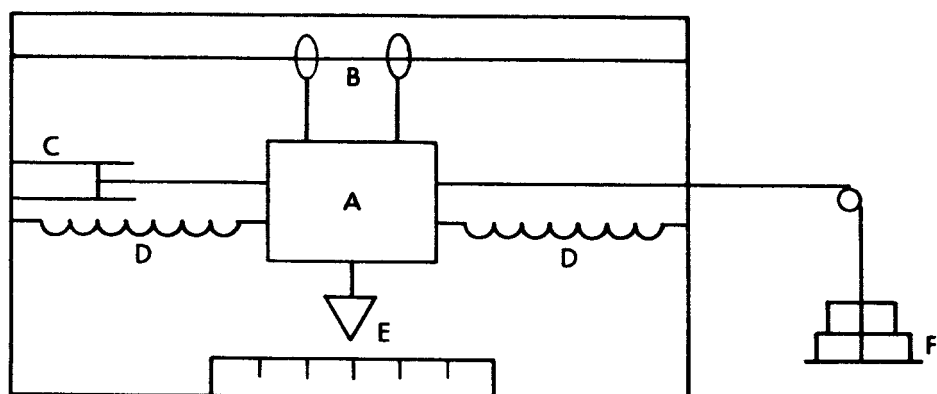
APPLICATIONS OF LLAMA

The interest in these very low accelerations is due to the possibility of using LLAMA as an antenna for the detection of gravitational waves (Ref. 4). The power spectral density of gravitational waves, if they exist in the universe, may be expected to increase with decreasing frequency, as the possible sources are double star systems, novae, etc. Thus the fact that LLAMA is tunable, in principle, to a natural period of several months makes the system an attractive candidate for this application.

However, the first application for the device is expected to be as the sensor in a dynamic test table leveling system - i. e. an active microseismic filter. The sensitivity required for this purpose is in the range 10^{-8} - 10^{-9} g.

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3. Ezekiel, S. , Towards a Low Level Accelerometer, M.I. T. Experimental Astronomy Laboratory Report TE-11, June, 1964.
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- A: TEST - MASS
- B: TEST - MASS SUSPENSION
- C: DAMPING
- D: RESTORING FORCE GENERATOR
- E: DISPLACEMENT READOUT
- F: CALIBRATION

Fig. 1 An elementary accelerometer.

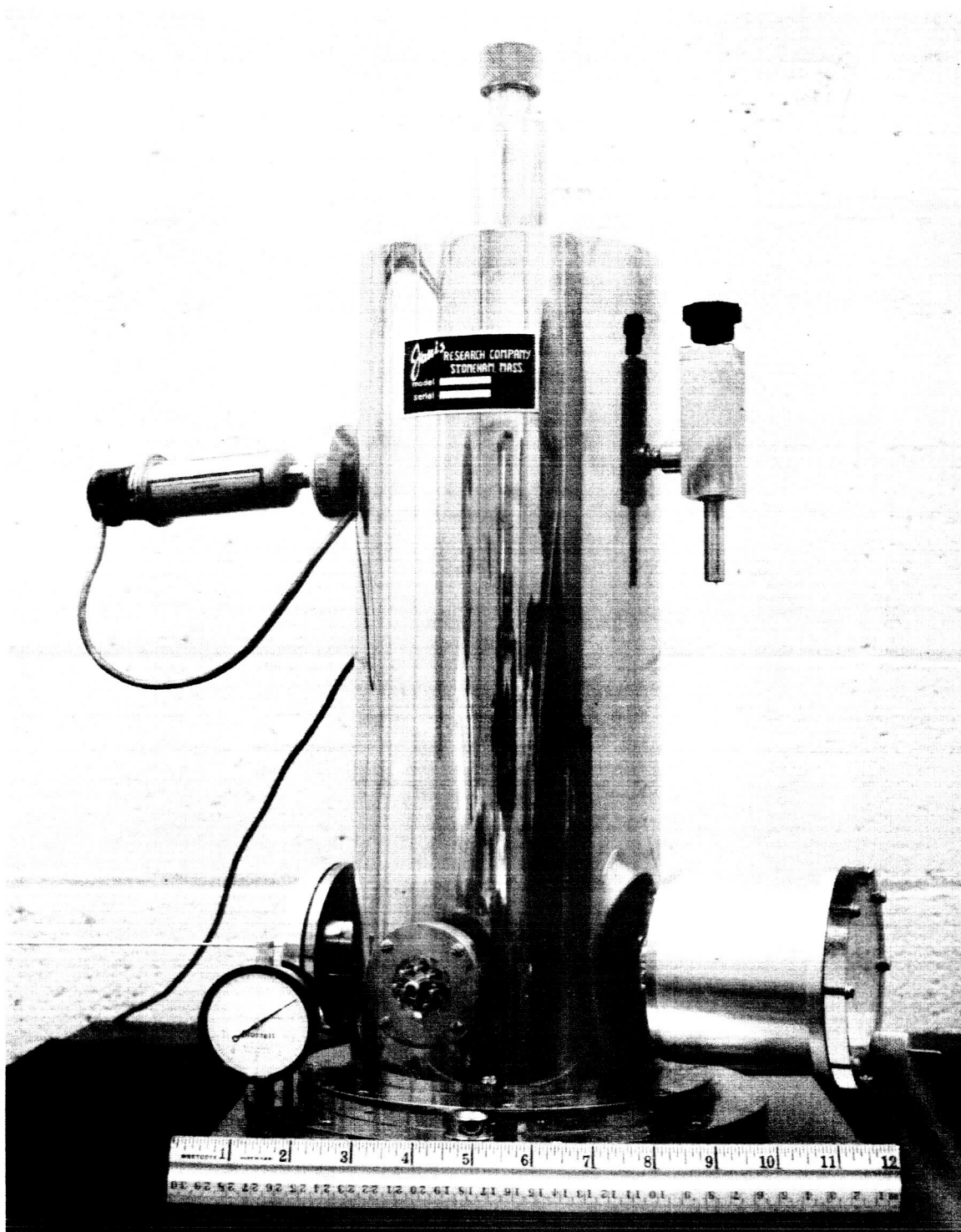


Fig. 2 The Llama Dewar.

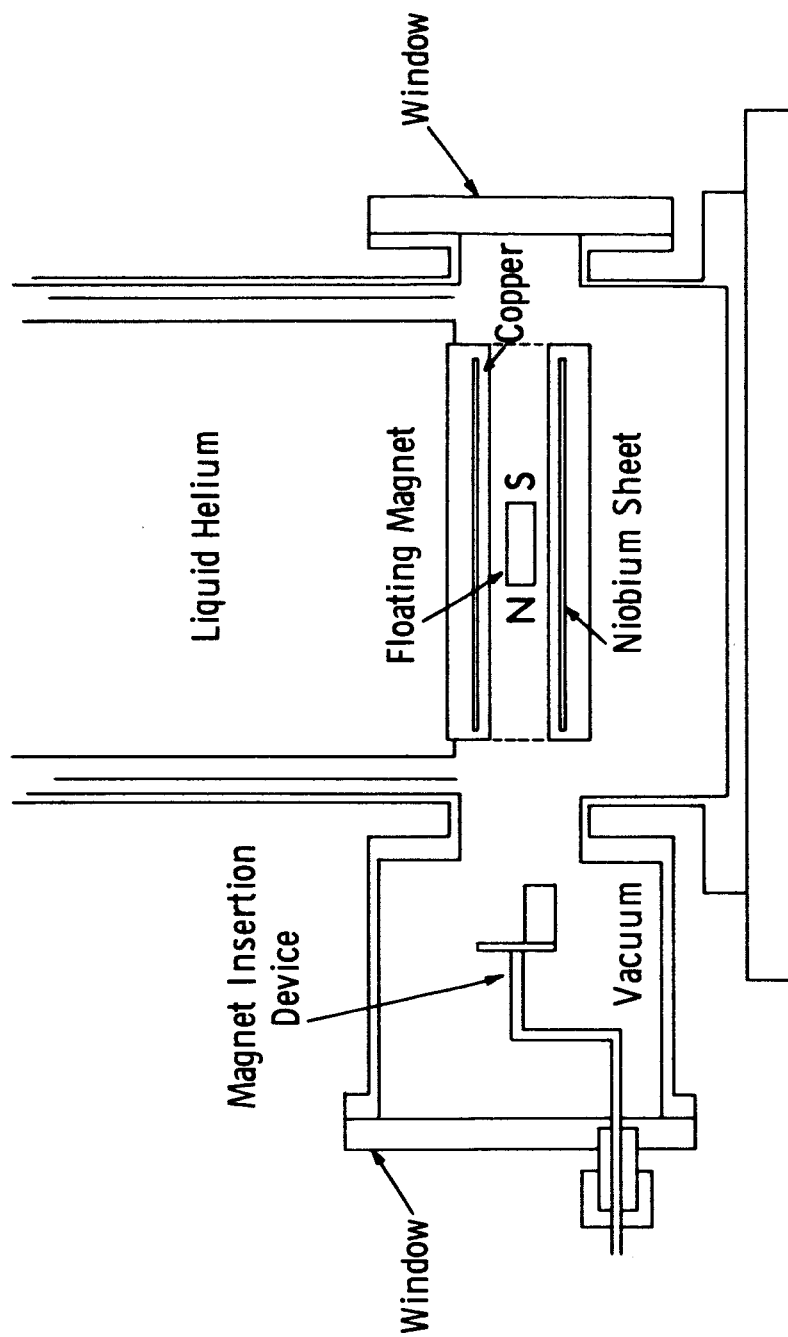


Fig. 3 Basic construction of metal dewar.

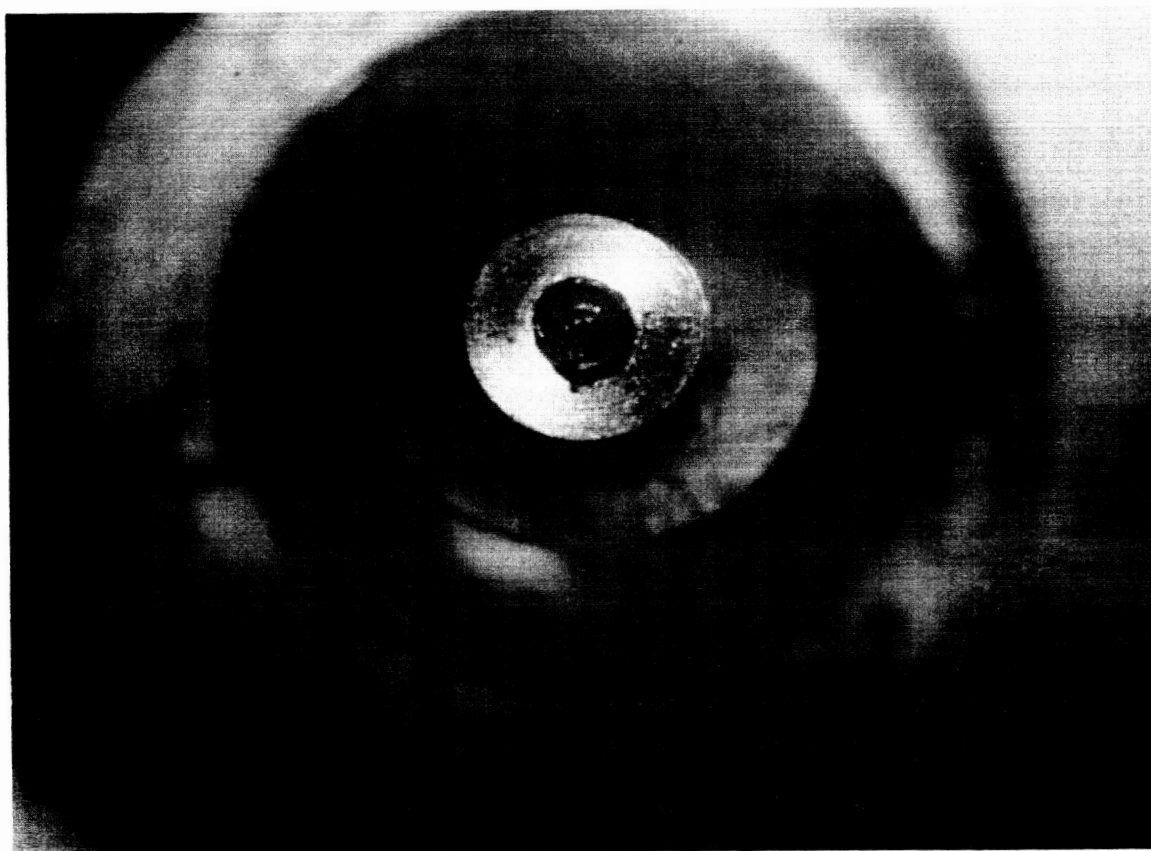


Fig. 4 Magnet floating in Llama suspension.

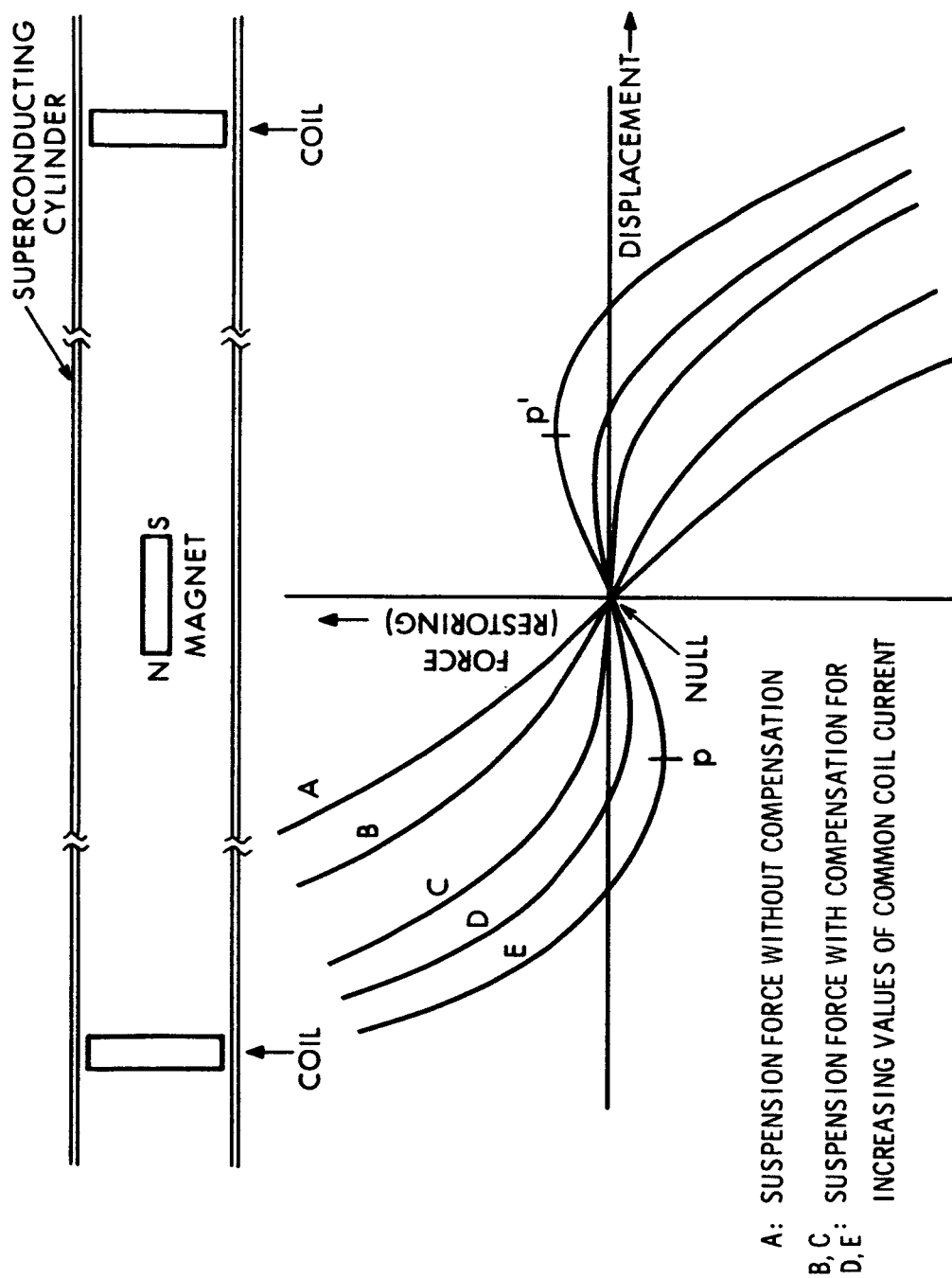


Fig. 5 Axial force on magnet close to null.

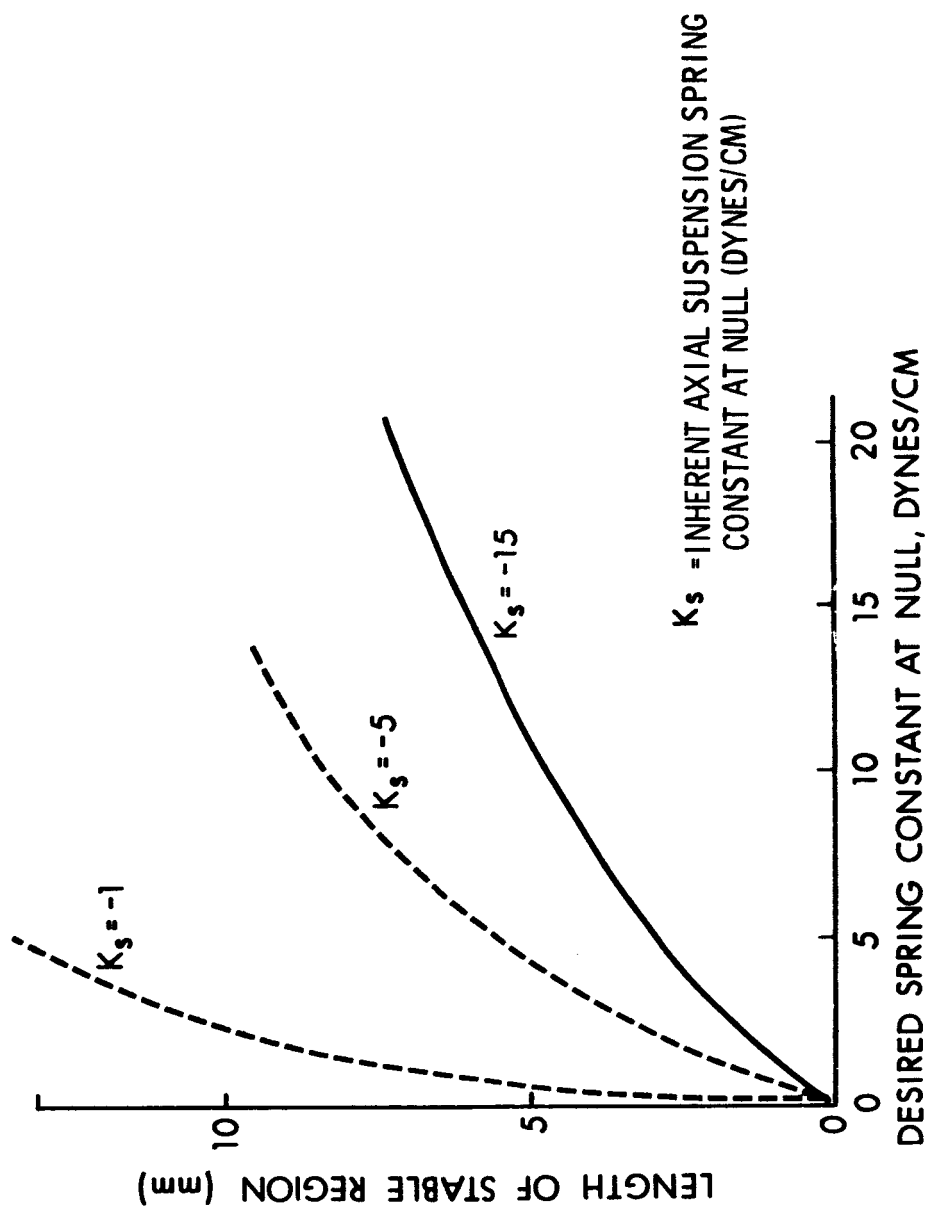


Fig. 6 Length of stable region vs desired spring constant at null.

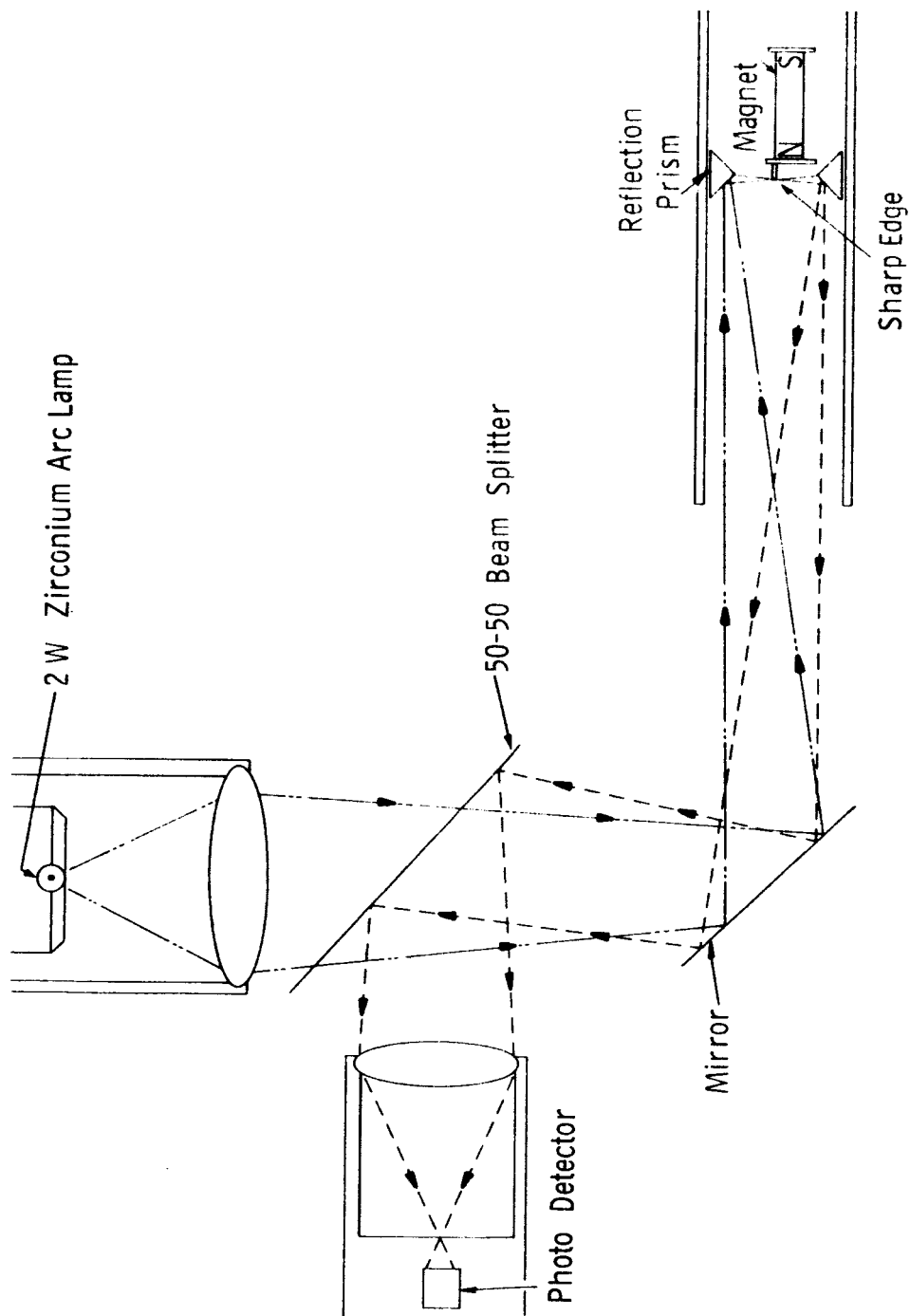


Fig. 7 Spot occultation displacement detector.

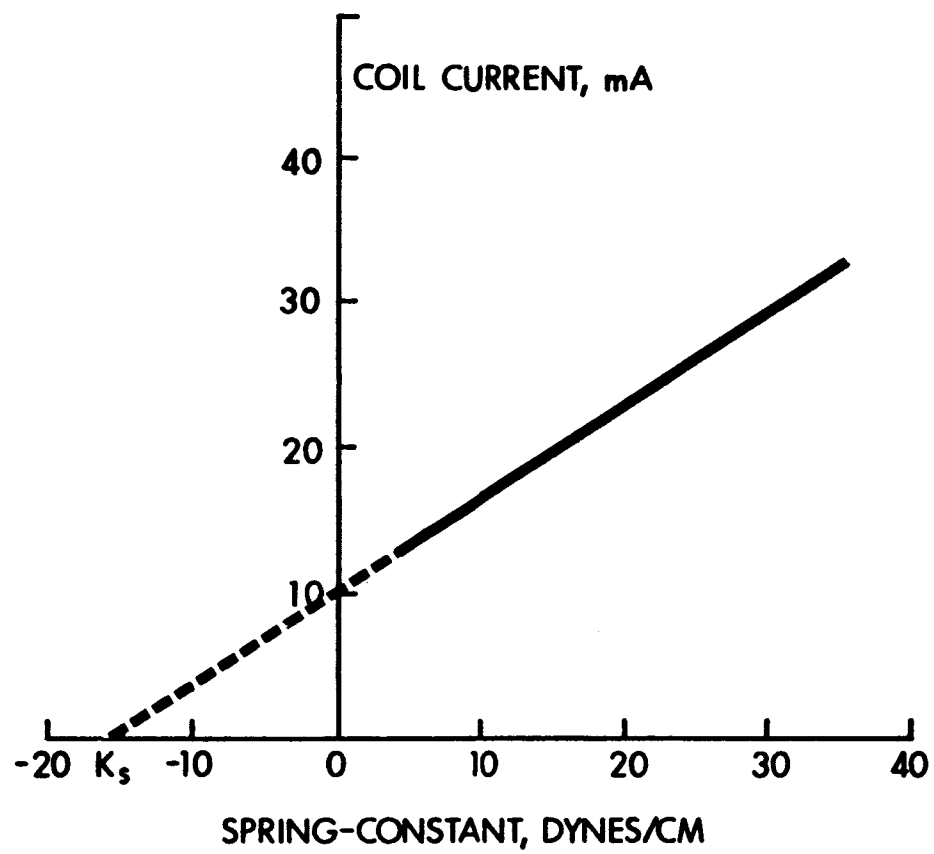


Fig. 8 Suspension axial spring constant vs coil current.

m	=	mass of magnet	τ_c	=	time constant of diff. mode force
k_f	=	suspension damping coefficient	τ_d	=	time constant of displacement detector
k_c	=	gain of coils	I	=	standing current in coils
k_a	=	gain of amplifier	θ	=	tilt error angle (small)
k_d	=	gain of displacement detector			

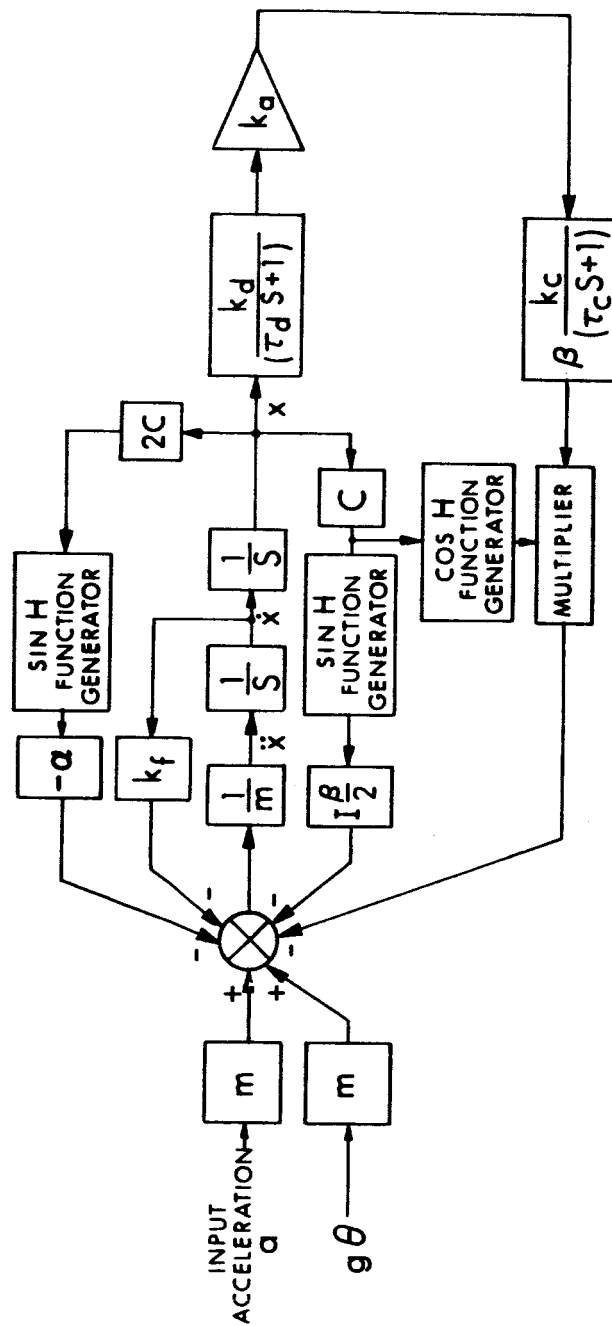


Fig. 9 Overall block diagram for accelerometer.

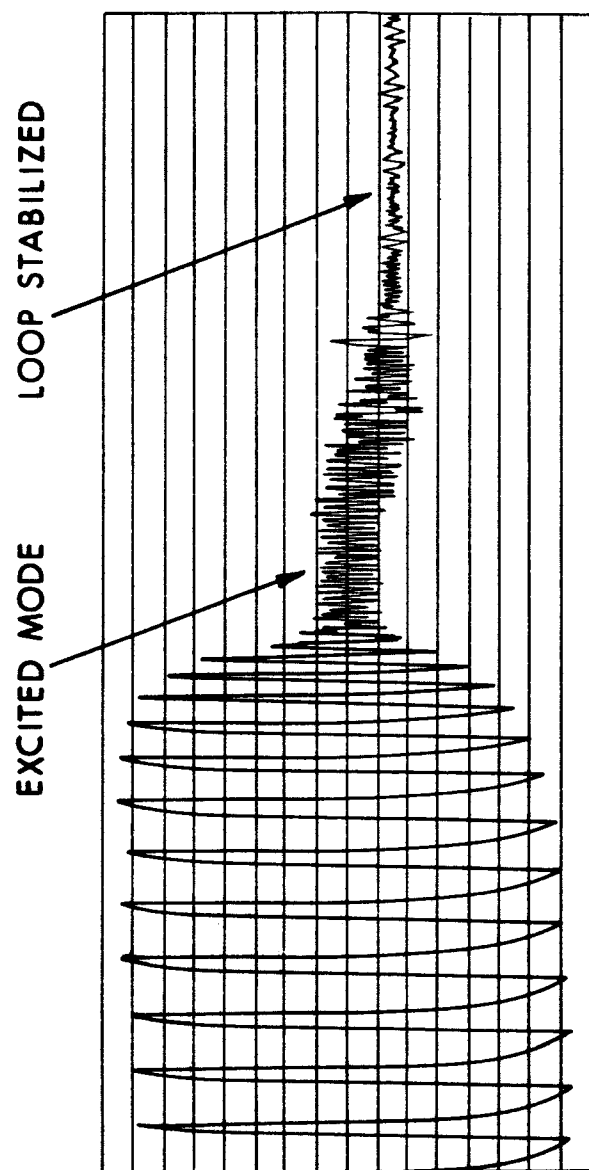


Fig. 10 Damping of closed loop oscillation.

$G_1(s)$	-	AXIAL DYNAMICS OF SUSPENSION	γ	=	CONSTANT: AXIAL FORCE/DIFFERENTIAL CURRENT
$H_1(s)$	-	ROTATIONAL DYNAMICS OF SUSPENSION	K	=	CONSTANT: TORQUE/DIFFERENTIAL CURRENT
$AG_2(s)$	-	GAIN AND DYNAMICS OF SERVO	u	=	NOISE
a_1	-	DETECTOR AXIAL SENSITIVITY	v	=	DISTURBANCE FROM ENVIRONMENT
a_2	-	DETECTOR ROTATIONAL SENSITIVITY			

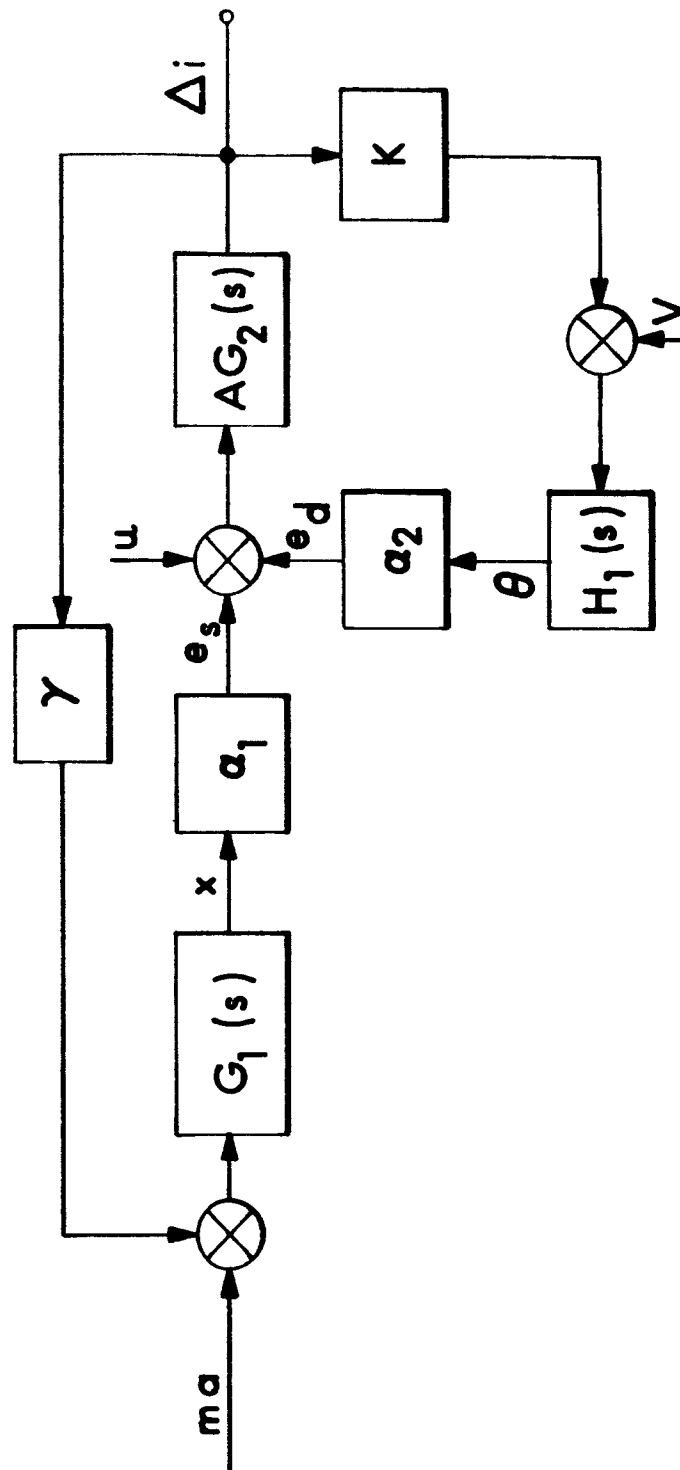


Fig. 11 Coupling of oscillatory modes.

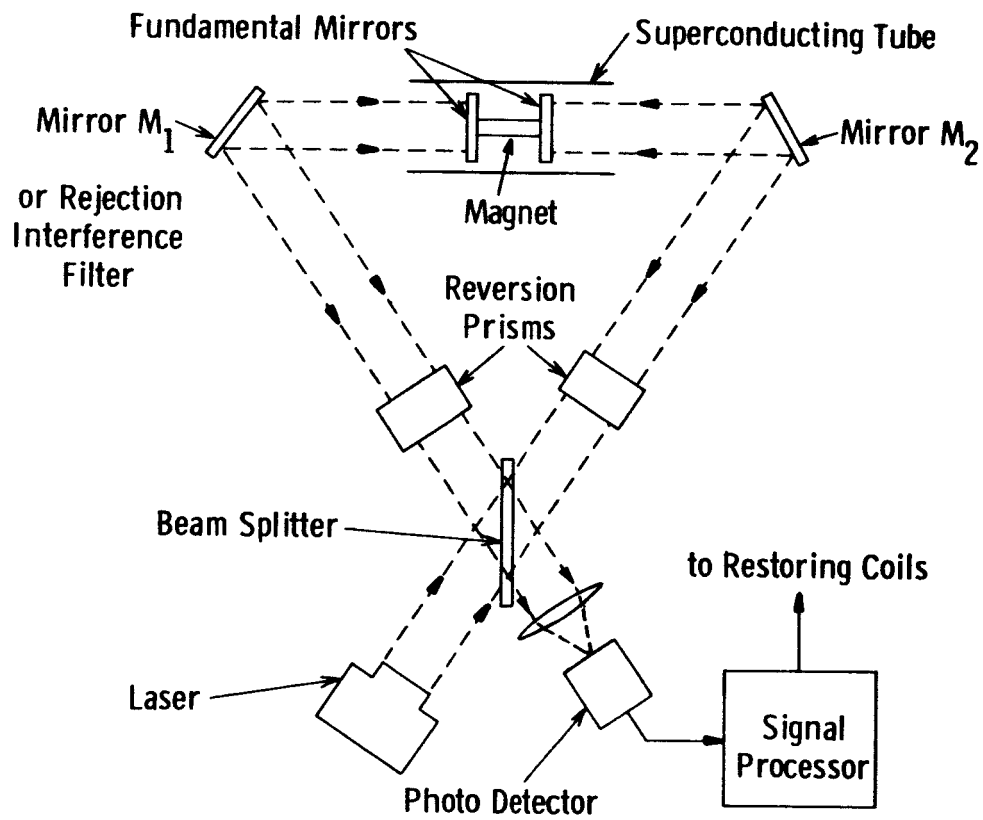


Fig. 12 Interferometer displacement detector.

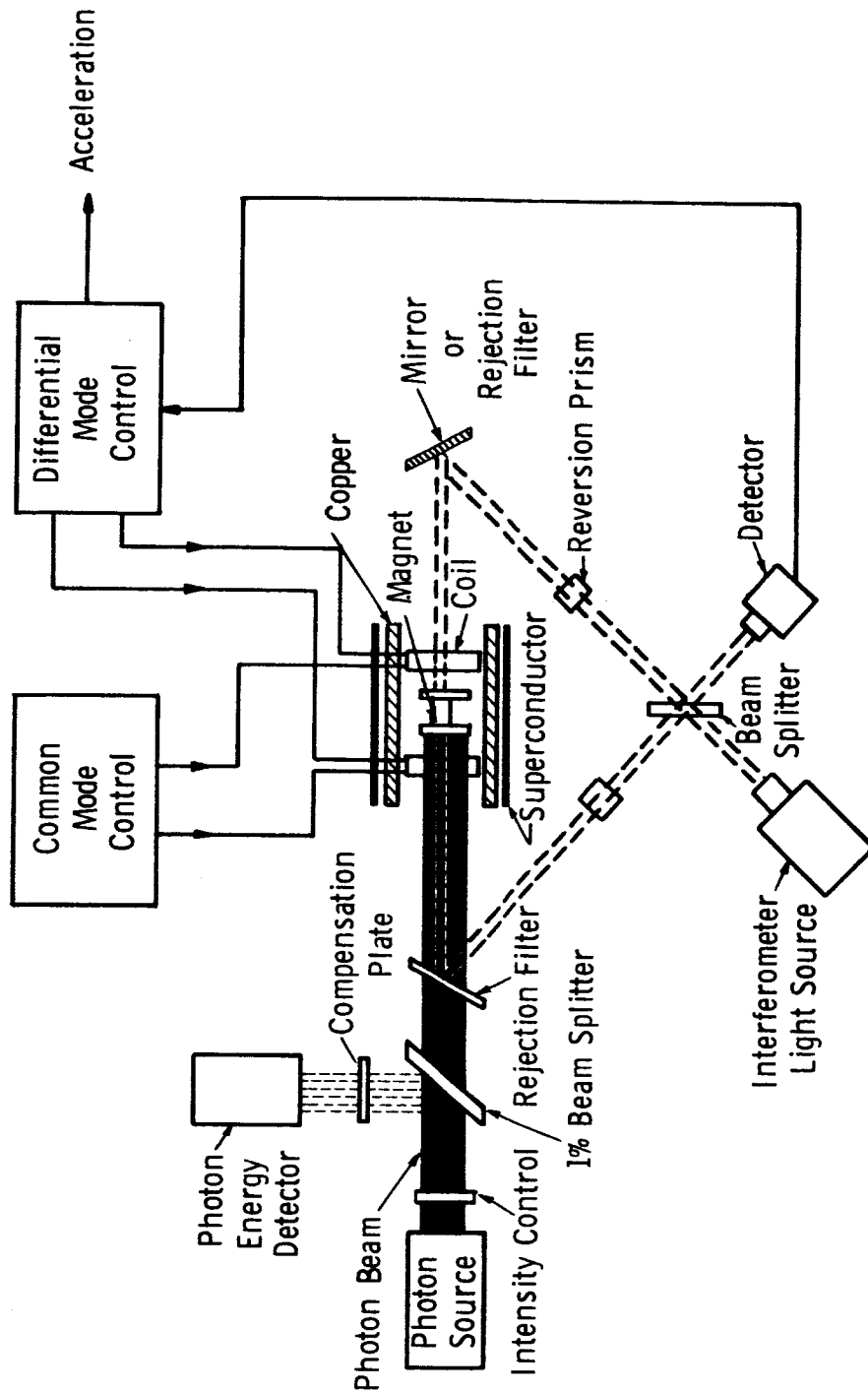


Fig. 13 The Llama system.